Mathematics 141                                             Final Exam

**Instructions**: Show all your work in your exam booklet.  A correct answer with little or no work to justify it may receive little or no credit.  Calculators are not permitted.

1.  Compute   $\int \dfrac{3x^2 + 5x - 4}{x^2 + x - 2} dx$

2.  Determine if  $\int_0^\infty \dfrac{dx}{\sqrt{1 + x^2}}$  converges (giving a reason for your answer), and if so, compute its value.

3.  A sample of radioactive material that at time $t = 0$ registered 810 counts per minute on a Geiger counter only produces 30 counts per minute at time $t = 10$ days.
    a.   What is the half-life of the substance?
    b.   When will the radioactivity be down to 1 count every 10 minutes?
    (You may leave the answers in terms of logarithms and exponentials)

4.  Determine the interval of convergence of $\displaystyle\sum_{n=1}^\infty \dfrac{nx^n}{2^n}$ .

5.  Find the volume of the solid obtained by revolving the region bounded by the curves
$$y = 2 + x^2 \quad \text{and} \quad y = 2x + 5$$
    around the x-axis.

6.  Determine if the limit exits, and if so, compute it:
$$\lim_{x \to 0^+} (\sin x)^{\tan x} .$$

7.  Compute $\int e^{2x} \cos 3x \, dx$ .

8.  Sketch the graph of the curve described in polar coordinates by the equation $r = 1 + 2\sin\theta$ , and find the area enclosed by the <u>inner</u> loop

9.   Determine whether or not the following series converge.  It is not necessary to compute the values of the convergent sums.  Explain carefully, stating what test(s) you are using.

    A. $\displaystyle\sum_{n=1}^\infty \dfrac{1 - \frac{1}{n}}{4n + 3}$ .    B. $\displaystyle\sum_{n=1}^\infty \dfrac{\sin n}{n!}$ .

10.  Let $f(x) = \sqrt{4 + x}$ , and consider Taylor's Theorem in the form $f(x) = p_1(x) + r_1(x)$, where $p_1(x)$ is the first-degree Taylor polynomial of f about x=0, and where $r_1(x)$ is the remainder.
    A.  Find $p_1(x)$, and by setting x=1, obtain an approximate value for $\sqrt{5}$
    B.  Use the formula for $r_1(x)$ to obtain an explicit upper bound on the error $|r_1(1)|$ in your approximation to $\sqrt{5}$.  You must justify your answer.
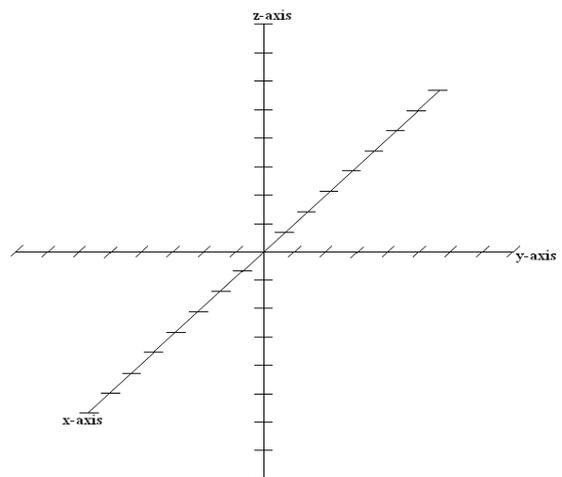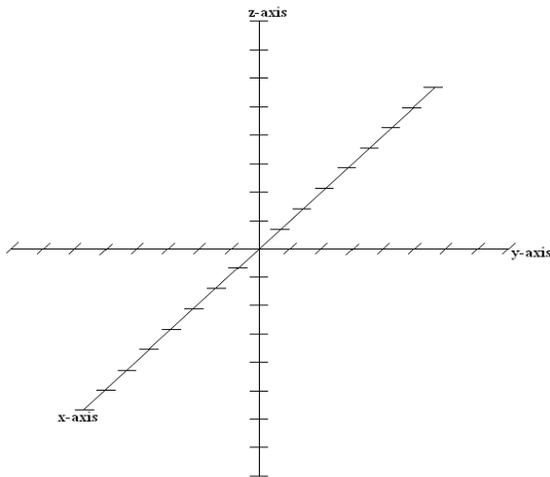
Vectors

These are straight off of Mr. Giles' Precalc B Exam Review, but since then, you may have only seen vectors in physics. Analysis II starts off heavy on vectors, and we do the review quickly. Remind yourself how basic operations (magnitude, products, equation forms) work for day 1.

1. Find the magnitude of the vector from (2, 6) to (10, –3).

2. Determine if the vectors below are parallel, perpendicular, or neither.
    a. <9, 6> and <2, –3>
    b. $\mathbf{w} = 10\mathbf{i} - 15\mathbf{j}$, $\mathbf{u} = -8\mathbf{i} + 12\mathbf{j}$

3. Write a vector equation and parametric equations of a line given the following:
    a. Line contains the point (2, 7) and is parallel to the vector <–3, 4>.
    b. Line contains the points (7, –3) and (4, 5).

Represent each of the following on the coordinate graphs below.
4. The position vector <–7, 6, 3>                     5. The plane $4x + 2y - z = 8 = 0$



6. Line L contains the points (4, –3, 8) and (–2, 4, –1).
    a. Write vector and parametric equations that represent L.
    b. Write the vector and parametric equation of a line parallel to L that contains (2, 0, –8).
    c. Write the equation of a line that is perpendicular to L.

7. Given points (2, 1, 0), (–1, 3, 5), and (4, 6, –2), use vectors to find the area of the triangle formed.

8. The point (3, 5, –6) is on a plane with normal vector $2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$. Write a Cartesian equation of the plane.

9. Write a Cartesian equation of the plane containing the points (1, 5, –2), (2, 6, –4), and (0, 5, 0) by finding a vector normal to the plane.